

## Design of Spur Gear

19.04.2019

Aim: (i) To determine dimensions of gear tooth [i.e.  $D = mZ$ ;  $b = \frac{4m}{\cos \phi} = 10m$ ,  $P_c = \pi m$ ; addl. =  $m$ ; ded =  $1.157m$ ]  
(ii) Module is determined by using beam strength equation of gear tooth.  
(iii) Checking the dimensions of the gear tooth w.r.t. bending and wear failure (i.e.  $F_{dynamic} \leq F_{beam \text{ strength}}$  &  $F_{dynamic} \leq F_{wear \text{ strength}}$ )

### \* Force Analysis:

For two mating or meshing gears:

- (i)  $\phi_1 = \phi_2 = \phi$  (Pressure angle is same)
- (ii)  $P_1 = P_2 = P$  (Power is same,  $\eta_{mech} = 100\%$ )
- (iii)  $F_{t_1} = F_{t_2} = F_t$
- (iv)  $m_1 = m_2 = m$ .
- (v)  $F_{R_1} = F_{R_2} = F_R$ .
- (vi)  $(F_R \text{ or } F_n) = \sqrt{F_t^2 + F_R^2} \therefore F_{R_1} = F_{R_2} = F_R$ .
- (vii)  $Z_1 \neq Z_2$ .
- (viii)  $D_1 \neq D_2$ .
- (ix)  $N_1 \neq N_2$
- (x)  $T_1 \neq T_2$ .

| Load          | $F_R$  | $F_t$  |
|---------------|--------|--|
| M/C component |        |  |
| Gear tooth    | A.C.L. | T.S.L  |
| Shaft         | T.S.L. | E.T.S.L ( $e=R$ ) = TSL + ( $T_M = F_t \times R$ ) |

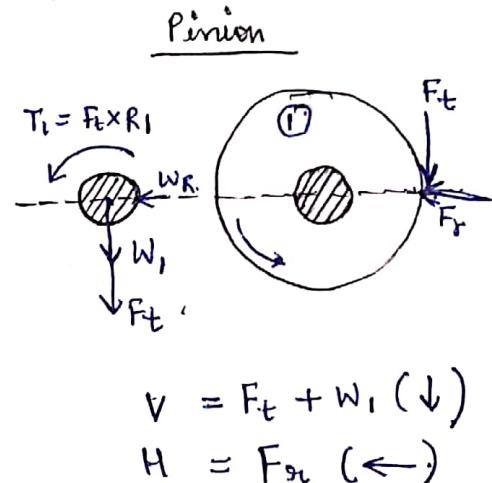
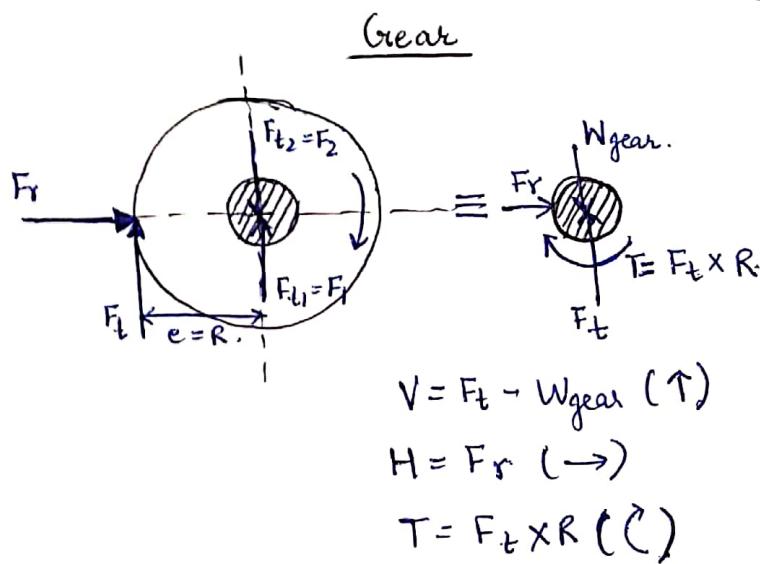
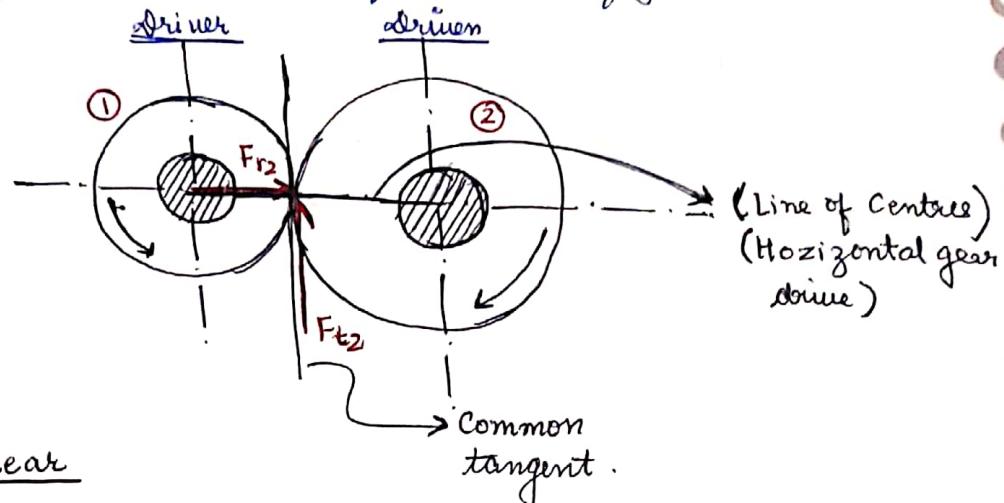
(i) Torque ( $T$ ) =  $\frac{60 \times 10^6 \times P}{2\pi N} = \text{_____ Nmm.}$

(ii)  $F_t = \frac{2T}{D}$

(iii)  $F_R = F_t \tan \phi$ .

(iv)  $F_R \text{ or } F_n = \sqrt{F_t^2 + F_R^2}$

- \*  $F_r$  will be always along the line of centre.
- \*  $F_t$  will be along the common tangent.
- \* W.r.t gear, the direction of  $F_r$  will in the direction of power transmission.
- \* Direction of  $F_t$  depends on direction of rotation of gear.



7 diagrams is drawn.:

- For TSL : One SFD, 1 BMD.
- For ETS : 1 SFD, 1 BMD
- Horizontal and Vertical Resultant diagrams
- Torque diagram.

Out of the above 7 diagrams, SFD in both cases is not drawn as shear stress on extreme fibres are zero and BM and TM are going to be maximum.

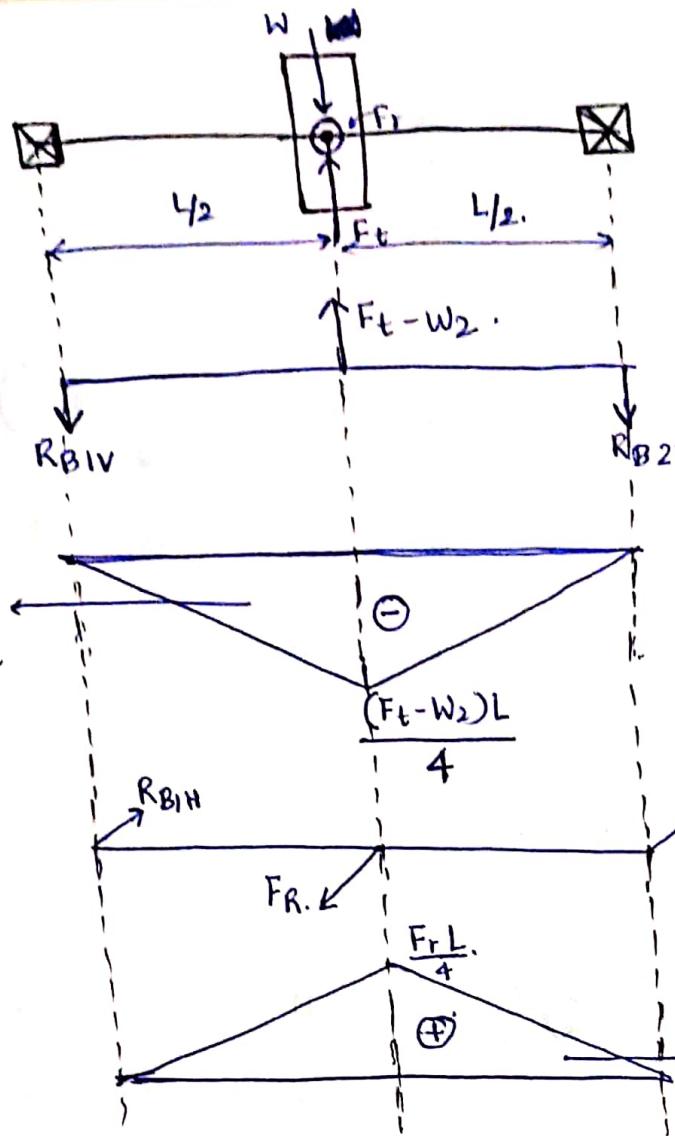
∴ Finally, there are 7 diagrams

•

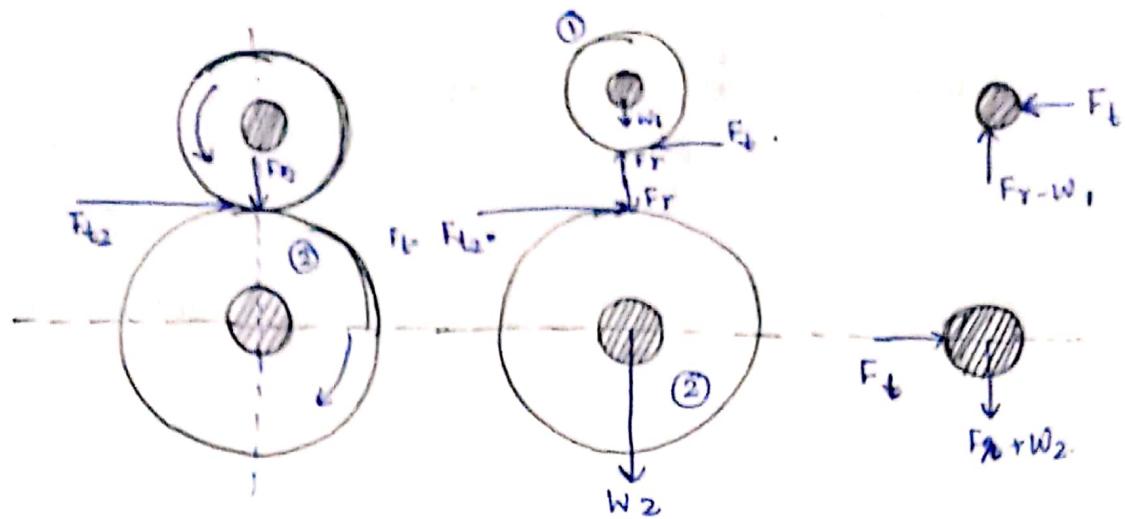
•

•

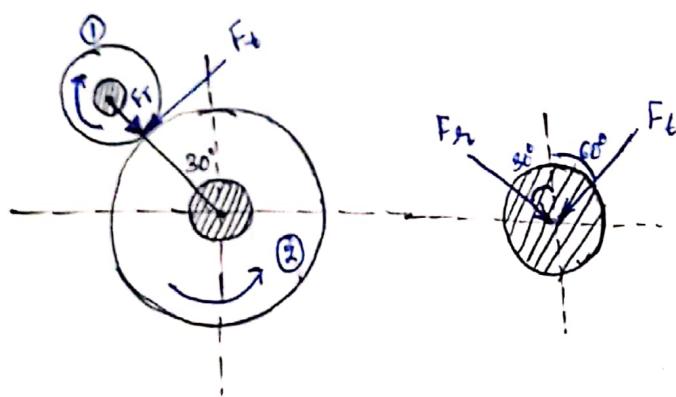
• Torque diagram.



- Short and square bearings
- Gear is mounted exactly at mid span.
- Gear rotation direction depends on  $F_t$
- Pinion rotation direction is decided by the driver shaft. It does not depend on  $F_t$ .



Vertical gear drive

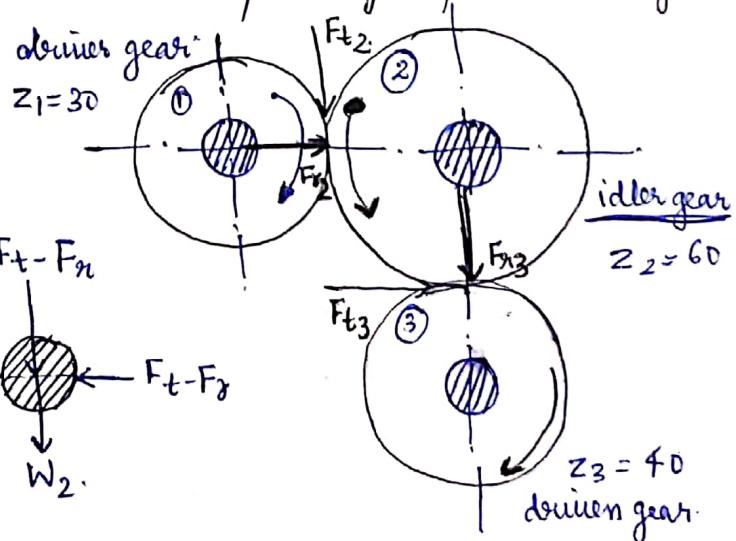
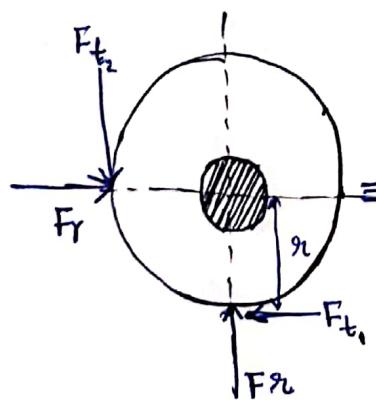


$$H = F_N \sin 30^\circ - F_t \sin 60^\circ$$

$$V = F_N \cos 30^\circ + F_t \cos 60^\circ + W_2$$

Q For the gear train as shown. Determine resultant force on idler gear shaft. Assume  $P = 3.5 \text{ kW}$  at  $700 \text{ rpm}$ . in CW direction, module,  $m = 5 \text{ mm}$ . no. of teeth on fronto driver gear, idler gear and driven gear are, 30, 60 and 40 respectively, pressure angle is  $20^\circ$

idler gear:



Net torque is zero  $[(F_t_1 \times r_1)_{\text{ACW}} + (F_t_2 \times r_2)_{\text{CW}}]$  hence, it's called idler

$$F_{t1} = F_{t2} = F_t$$

$$F_{t2} \text{ & } F_t \Rightarrow T_1 = F_t \times R \text{ (cw)}$$

$$F_{t1} = F_t \Rightarrow HTSL (\leftarrow)$$

$$F_{t3} = F_{t4} = F_t$$

$$F_t \text{ and } F_{t4} \Rightarrow T_2 = F_t \times R \text{ (Acw)}$$

$$F_{t3} = F_t \Rightarrow VTS \text{ (}\downarrow\text{)}$$

$$\therefore T = T_1 + T_2 = 0$$

$$P_2 = T\omega = 0$$

∴ Idler shaft is designed either ~~by~~ using bending equation or strength criterion. [theories of failures are optional]

\* For idler gear shaft:

$$H = F_t - w_2 \text{ (}\leftarrow\text{)}$$

$$V = F_t + w_2 - F_y \text{ (}\downarrow\text{)} = F_t - F_y \text{ (}\downarrow\text{) neglecting weight}$$

$$R = \sqrt{H^2 + V^2} = \sqrt{2} H = \sqrt{2} V = \sqrt{2} (F_t - F_y)$$

$$\textcircled{1} \quad T_1 = \frac{60 \times 10^6 P}{2\pi N_1} = 47746.48293 \text{ N-mm.}$$

$$\textcircled{2} \quad T_2 = 0$$

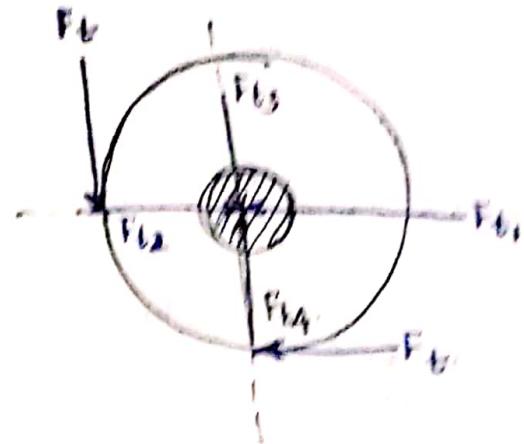
$$\textcircled{3} \quad T_3 = \frac{60 \times 10^6 P}{2\pi N_3} \text{ or } \frac{T_3}{T_1} = \frac{z_3}{z_1} \text{ (for } \eta_m = 100\%) \quad T_3 = 63661.37724 \text{ N-mm}$$

$$\textcircled{4} \quad F_{t3} = \frac{2T_1}{D_1} \text{ or } \frac{2T_3}{D_3} = 636.6198 \text{ N.}$$

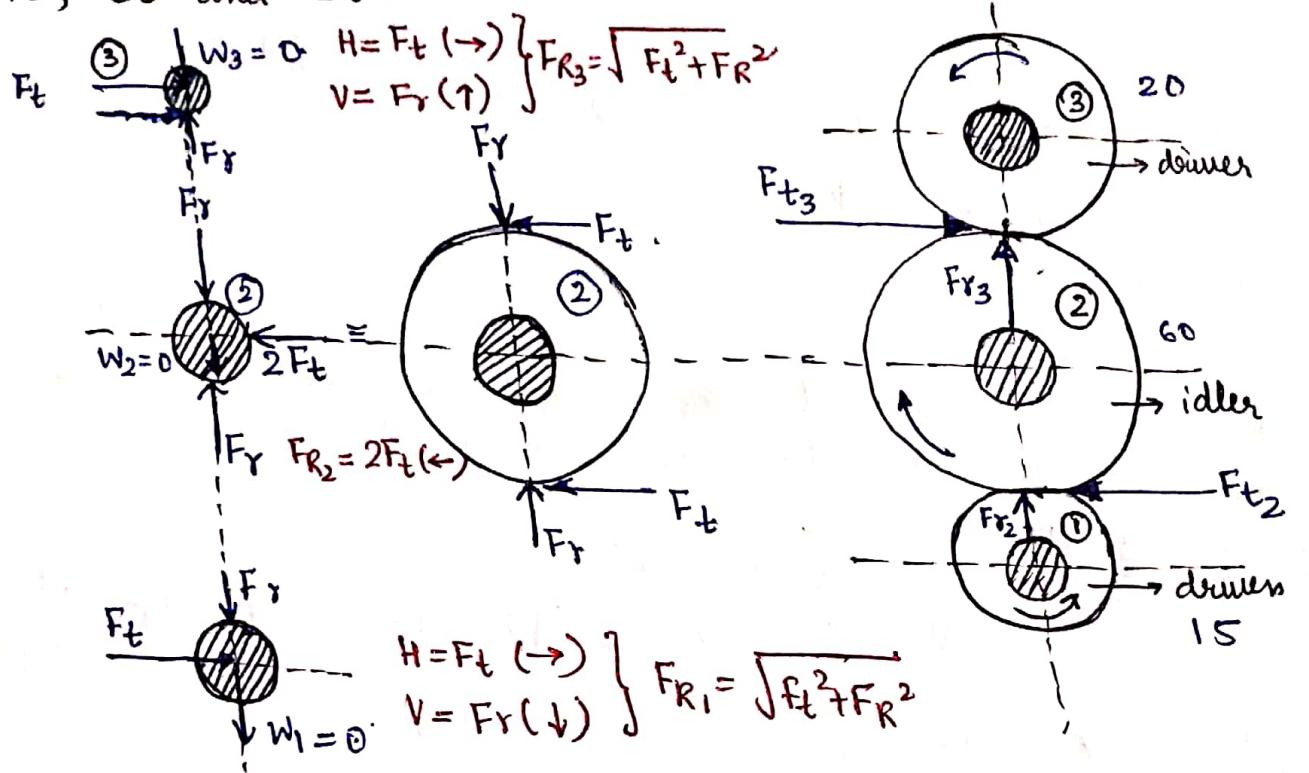
$$\textcircled{5} \quad F_R = F_t \tan \phi = 231.7106 \text{ N.}$$

$$\textcircled{6} \quad (F_R)_{\frac{1}{2}} = \sqrt{2} (F_t - F_y) = 572.628 \text{ N.}$$

$$\textcircled{7} \quad F_{R1} = F_{R3} = \sqrt{F_t^2 + F_R^2} = 677.47 \text{ N. (by neglecting weight)}$$



② For gear train as shown in Fig. determine resultant forces on driver shaft, idler gear shaft and driven shaft.  $P = 10 \text{ kW}$  at  $500 \text{ rpm}$   
 module =  $5 \text{ mm}$ ;  $\phi = 20^\circ$ ; no. of teeth on driver, idler and driven are 15, 60 and 20.



$$① T_1 = \frac{60 \times 10^6 \times P_0}{2 \pi N_1} = 190985.9317 \text{ N-mm}$$

$$② T_2 = 0$$

$$③ \frac{T_3}{T_1} = \frac{z_3}{z_1} = 254647.9009 \text{ N-mm}$$

$$④ F_t = \frac{2T_1}{D_1} \text{ or } \frac{2T_3}{D_3} = 5092.958 \text{ N}$$

$$⑤ F_y = F_t \tan \phi = 1853.685 \text{ N}$$

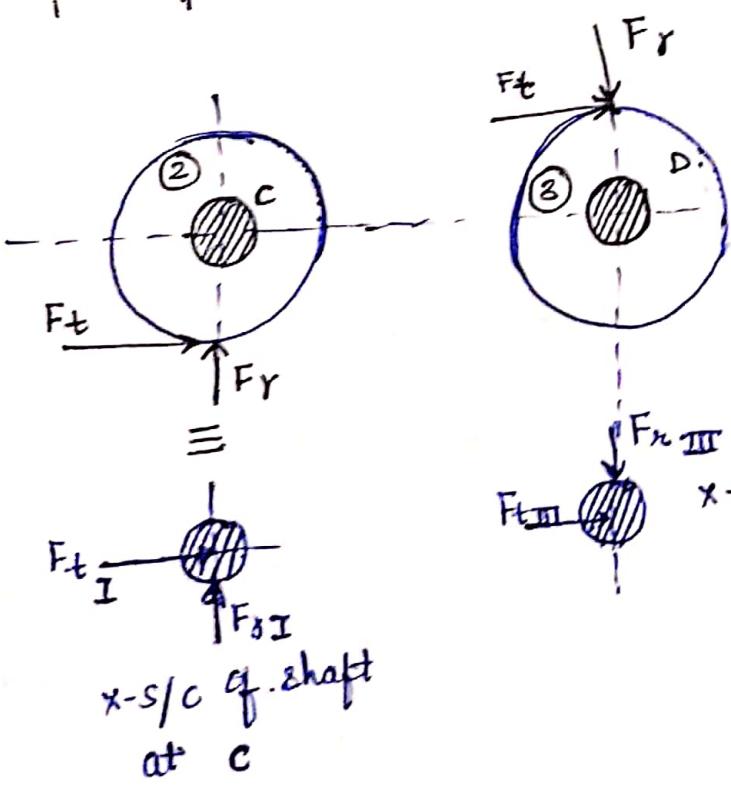
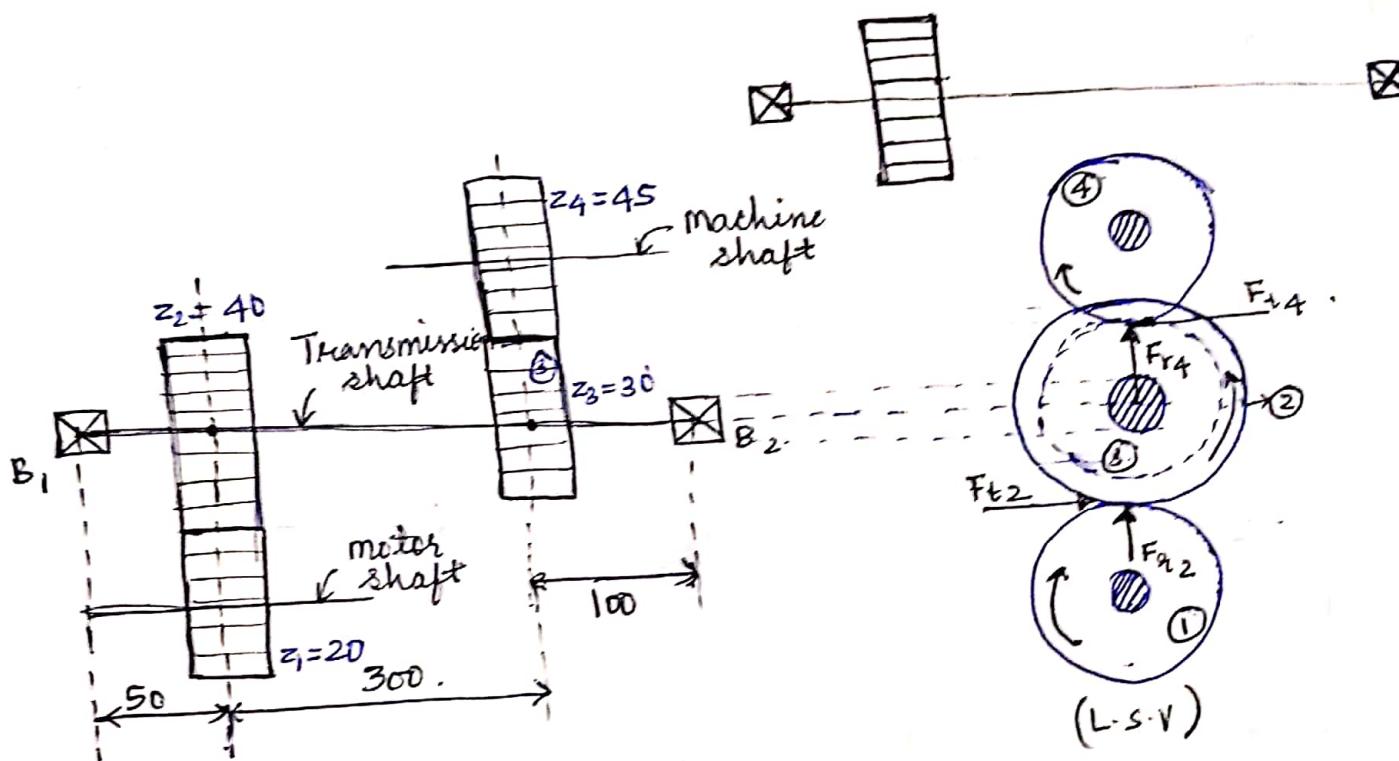
$$⑥ F_{R1} = F_{R3} = \sqrt{F_t^2 + F_R^2} = 5419.812 \text{ N}$$

$$⑦ F_{R2} = \sqrt{2F_t^2} = 10185.916 \text{ N}$$

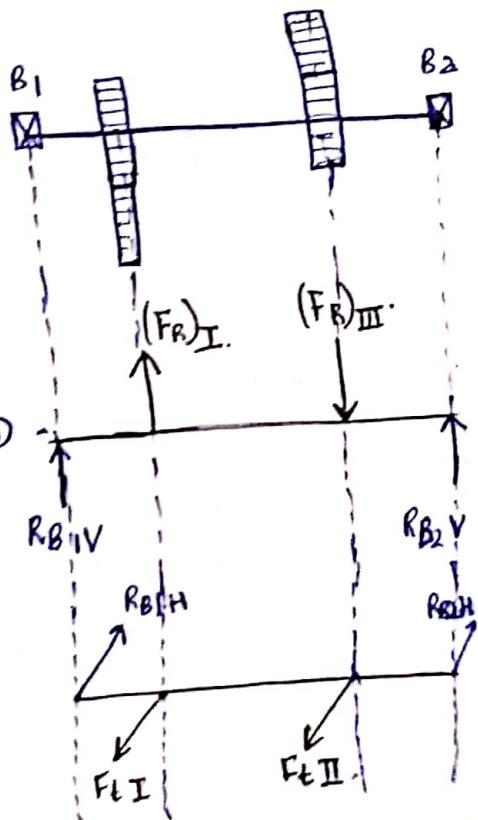
Q Determine Reactions on bearing supporting the shaft upon which gears 2 and 3 are mounted as shown in Fig. Gear 1 is driver with 20 teeth, 6mm module, 20° full depth gear, rotating at 2000 rpm in CW direction and transmitting 80 kW.

Gear 2: 40 teeth      Gear 3: 5mm module 20° F.D. with 30 teeth.  
Gear 4 has 45 teeth

So



x-s/c of shaft at D



$$T_1 = \frac{60 \times 10^6 \times \rho}{2 \times N_1} = 381.9718 \times 10^3 \text{ N-mm.}$$

$$(F_t)_I = \frac{2T_1}{D_1} = \frac{2T_1}{20 \times 6} = 6366.1977 \text{ N}$$

$$(F_{\text{R}})_I = (F_t)_I \tan \phi = 2317.1064 \text{ N}$$

$$\frac{T_2}{T_1} = \frac{z_2}{z_1} \Rightarrow T_2 = 763943.6 \text{ N-mm} = T_3 \quad \left( \begin{matrix} P_3 = P_2 \\ N_3 = N_2 \end{matrix} \right)$$

$$(F_t)_{\text{II}} = \frac{2T_3}{D_3} = 10185.8147 \text{ N}$$

$$(F_{\text{R}})_{\text{III}} = (F_t)_{\text{III}} \tan \phi_{\text{II}} = 3707.3697 \text{ N}$$

$$F_{t1} = F_{t2} = F_{t\text{I}} \quad ; \quad F_{\text{R}1} = F_{\text{R}2} = F_{\text{R}\text{I}}$$

$$F_{t3} = F_{t4} = F_{t\text{III}} \quad F_{\text{R}3} = F_{\text{R}4} = F_{\text{R}\text{III}}$$

$$R_{B1V} = \frac{(-2317.106)(400) + (3707.36) \times 100}{450} = -1235.792 \text{ N}$$

or  $R_{B1V} = 1235.792 \text{ N} (\downarrow)$

$$R_{B2V} = +1235.792 + -2317.106 + 3707.36 = 2626.046 \text{ N}$$

$$R_{B1H} = \frac{(6366.1977 \times 400) + (10185.89 \times 100)}{450} = 7922.373 \text{ N} (\uparrow)$$

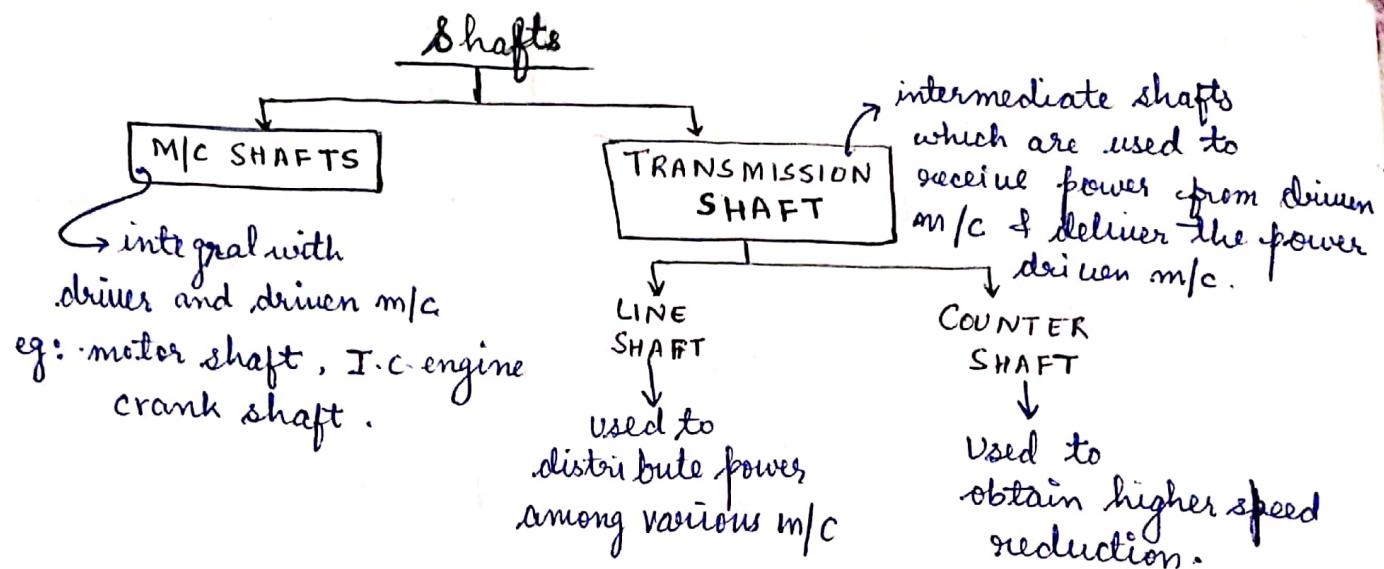
$$R_{B2H} = 8629.71 (\uparrow)$$

∴ Resultant reaction on bearing:

$$R_{B1R} = \sqrt{(R_{B1V})^2 + (R_{B1H})^2} = 8018.176 \text{ N}$$

$$R_{B2R} = \sqrt{(R_{B2V})^2 + (R_{B2H})^2} = 9020.4 \text{ N}$$

- \* Journal bearing is used if speed is higher and shaft works continuously
- \* (Cylindrical roller) bearing <sup>Anti-friction</sup> if low speed and intermittent operation
- \* Transmission shaft →
  - line shaft (used when more than <sup>one</sup> machine is to operate with <sup>single</sup> motor)
  - counter shaft (here in this case, the transmission shaft is counter one) used when one machine is to operate [run 2 gear drives]



### ★ Design procedure used in spur gears :-

#### Input Data:

1.  $P = \pi kW$  at  $\gamma \text{ rpm}$
2. Pressure angle ( $\phi$ ) = \_\_\_\_\_
3. Gear ratio ( $G_1$ ) = \_\_\_\_\_
4.  $[\sigma_b] = \text{per bending stress} = \frac{\text{failure stress}}{N} \text{ ( } N = 3 \text{ default) }$
5. Lewis form factor ( $y$ ) = \_\_\_\_\_
6.  $C_v = \text{Velocity factor} = _____$
7.  $\sigma_{es} = \text{surface endurance limit} = (2.8 \times \text{BHN} - 70) \text{ MPa}$
8.  $E = \text{young's modulus}$

#### Optional:

9. No. of teeth on pinion ( $z_1$ ) (by using Rack & pinion formula)
10.  $\Psi = b/m = 8 \text{ to } 14$  ( default = 10 )
11. Service factor ( $C_s$ ) = 1.2 to 1.3 default

Steps: Subscripts 1 and 2 for pinion and gear respectively -

1)  $T_1 = \text{torque to be transmitted by pinion}$ .

$$(\text{Mean torque}) T_1 = \frac{P \times 60 \times 10^6}{2\pi N_1} = \text{_____ N-mm.}$$

2)  $[T_1] = \text{design torque for pinion} = T_1 \times C_s \text{ (max. torque)}$

3)  $z_1 \geq (z_1)_{\min}$  where  $(z_1)_{\min} = \frac{2 A_W}{\sin^2 \phi}$  for F.D,  $A_W = 1$ .  
 for stub tooth,  $A_W = 0.8$

4)  $z_2 = G_1 z_1$

5) module (m) :- module is determined from following eqn which is obtained from beam strength of weaker gear (W.G.)

$$m \geq 1.26 \sqrt[3]{\frac{[T_1]}{([\sigma_b]Y)_{W.G.} \Psi z_1}}$$

Q) \*\* when gear and pinion are made of same material, pinion is the weaker gear

$$\because [\sigma_{b1}] = [\sigma_b]_2 \therefore b_1 = b_2 = b; m_1 = m_2 = m;$$

$$Y_1 < Y_2 \Rightarrow [F_s]_1 < [F_s]_2.$$

$$\text{Hence, } ([\sigma_b]Y)_{W.G.} = [\sigma_b]Y.$$

\*\* when they are made of different materials.,

$$([\sigma_b]Y)_{W.G.} = \min \text{ of } [\sigma_{b1}]Y_1 \text{ and } [\sigma_{b2}]Y_2.$$

Q)  $Y = \pi y$ .

$$\therefore m \geq \text{_____ mm.}$$

6) dimensions of gear train :-

$$D_1 = m z_1 = \text{_____ mm.}$$

$$D_2 = m z_2 = \text{_____ mm.}$$

$$b = \Psi m = 10m = \text{_____ mm.}$$

7)  $F_s$  = beam strength of W.G.

$$F_s = ([\sigma_b]Y)_{W.G.} b m = \text{_____ N.}$$

8)  $F_d$  = dynamic load.

I Method:

$$(i) F_d = \frac{2 [T_1]}{D_1} \text{ or } \frac{2 [T_2]}{D_2}$$

$$(ii) V_1 = V_2 = V = \frac{\pi D_1 N_1}{60} \text{ or } \frac{\pi D_2 N_2}{60} = \text{_____ m/s}$$

$$(iii) C_V = \frac{3+v}{3} \text{ or } \frac{3}{3+v} \text{ (if } v \leq 10 \text{ m/s)} .$$

$$(iv) F_d = F_t \times C_V \quad \text{or} \quad \frac{F_t}{C_V} \rightarrow C_V < 1. = \text{--- N.}$$

if  $F_d \leq F_s \Rightarrow$  design is safe w.r.t. bending failure.

II method: (Buckingham eq<sup>n</sup>):

$$F_d = F_t + f_i \leftarrow \text{incremental load.}$$

$$F_i = \frac{21V(Cb + F_t)}{21V + \sqrt{Cb + F_t}}, \text{ where } c = \text{dynamic load const in N/mm}$$

$$c = \frac{Ke}{\left[ \frac{1}{E_1} + \frac{1}{E_2} \right]} \text{ mm.}$$

$e$  = error in tooth action in mm.

$K = 0.11$  for  $\phi = 14\frac{1}{2}^\circ$

$K = 0.114$  for  $\phi = 20^\circ$

### 9) Wear strength (for pinion only)

$$F_w = D_1 Q k b \text{ in N.}, \text{ where } Q = \frac{2G}{G \pm 1} \quad (+) \text{ for external gears}$$

$(-)$  for internal gears

$$F_d \leq F_w$$

\* Design is safe w.r.t. wear failure.

$k$  = material combination factor.

$$k = \frac{(c_{es})^2 \sin \phi \left[ \frac{1}{E_p} + \frac{1}{E_G} \right]}{1.4} \text{ MPa.}$$

$$k = 0.16 \left[ \frac{BHN}{100} \right]^2 \text{ when both gear + pinion are of STEEL}$$

Abrasive : foreign particle

corrosive : chemical action of lubricant

scoring : metal to metal contact.

fatigue : due to repeated loading (fatigue.)

Q A pair of spur gears having 20 full depth involute teeth is to transmit 20 kW. The pinion runs at 300 rpm and speed ratio 3:1. The following data are given. No. of teeth on pinion is 15. Service factor = 1. Velocity factor =  $\frac{3}{3+v}$ , tooth form factor ( $y$ ) =  $[0.154 - \frac{0.912}{z}]$ . Face width,  $b = 14\text{m}$ , allowable elastic stress for pinion and gear material are 120 MPa and 100 MPa respectively. Surface endurance limit 600 MPa. Young's modulus of pinion: 200 GPa;  $E_1 = 100\text{ GPa}$ . Design the gear tooth and check for bending and wear failure.

SoP<sup>m</sup>:

- 1)  $P = 20\text{ kW}$  at 300 rpm: 4) No. of teeth on pinion,  $z_1 = 15$ .
- 2) Pressure ratio,  $\phi = 20 \cdot \text{F.D.}$  5) Service factor,  $C_S = 1$ .
- 3) Gear ratio,  $G_1 = 3$ . 6) Velocity factor,  $C_V = \frac{3}{3+v}$
- 7)  $y = \text{tooth form factor} = 0.154 - \frac{0.912}{z}$
- 8) face width = 14 m  $\Rightarrow \phi = \frac{b}{m} = 14$ .
- 9) Permissible elastic stress for pinion  $[\sigma_b]_1 = 120\text{ MPa}$ .
- 10) " " " " " gear  $[\sigma_b]_2 = 100\text{ MPa}$
- 11) Surface <sup>endurance</sup> elastic limit =  $K = \sigma_{eS} = 600\text{ MPa}$ .
- 12)  $E_1 = 200\text{ GPa}$ ;  $E_2 = 100\text{ GPa}$ .

Steps: Subscripts 1 and 2 represents pinion and gear respectively.

- 1)  $T_1 = \text{torque to be transmitted by pinion} = \frac{P \times 60 \times 10^6}{2\pi N_1} = 636619.7724\text{ N-mm}$
- 2) Design torque  $[T_1] = C_S \times T_1 = 636619.7724\text{ N-mm}$  for pinion.
- 3)  $z_2 = \text{no. of teeth on gear} = G_1 z_1 = 45$ .
- 4) Finding the weaker gear [Finding module].

$$[\sigma_b]_1 Y_1 = 120 \times \left[ 0.154 - \frac{0.912}{z_1} \right] = 35.1355 \text{ MPa}$$

$$[\sigma_b]_2 Y_2 = 100 \times \left[ 0.154 - \frac{0.912}{z_2} \right] = 42.0136 \text{ MPa}$$

$\therefore [\sigma_b]_1 Y_1 < [\sigma_b]_2 Y_2 \therefore$  pinion is the weaker wheel.

$$([\sigma_b]_1 Y_1)_{wg} = [\sigma_b]_1 Y_1 = 35.135 \text{ MPa}$$

Hence, module should be calculated w.r.t. beam strength of pinion.

$$m \geq 1.26 \sqrt{\frac{T_1}{[(\sigma_b)Y]_{W_6} \times \Psi \times Z_1}}, \quad m \geq 5.5676 \text{ mm} \quad \text{or} \\ m \geq 6 \text{ mm.}$$

5) Dimensions of gear train

$$D_1 = mZ_1 = 90 \text{ mm}; \quad D_2 = mZ_2 = 270 \text{ mm}, \quad b = 14 \text{ m} = 24 \text{ mm.}$$

6)  $F_s$  = beam strength of W6

$$F_s = ([(\sigma_b)Y]_{W_6} b) m = 17702.292 \text{ N.}$$

7)  $F_d$  = dynamic load:

$$F_d = \frac{2[T_1]}{D_1} = 14147.10605 \text{ N}$$

$$V = \frac{\pi D_1 N_1}{60} = 1.4137 \text{ m/s.}$$

$$C_V = \frac{3}{3+V} = 0.6737.$$

$$F_d = \frac{F_d}{C_V} = 20813.7727 \text{ N.}$$

$$\therefore F_d > F_s$$

∴ Design is unsafe w.r.t. bending failure.

Hence, new module is calculated using by equating  $F_d = F_s$ .

8) New module:

$$F_s = F_d = 20813.7727 = ([(\sigma_b)Y]_{W_6} b) m = (35.1355) \times 14 \text{ m} \times m.$$

$$m = 6.5 \approx 7 \text{ mm.}$$

9) Dimensions of gear train (new revised)

$$D_1 = mZ_1 = 105 \text{ mm}; \quad D_2 = mZ_2 = 315 \text{ mm}, \quad b = 98 \text{ mm.}$$

$$F_s = ([(\sigma_b)Y]_{W_6} b) m = 24102.953 \text{ N.}$$

10)  $F_d$  = dynamic load:

$$F_d = \frac{2[T_1]}{D_1} = 12126.0909 \text{ N.}$$

$$V = \frac{\pi D_1 N_1}{60} = 1.6493 \text{ m/s.}$$

$$C_V = \frac{3}{3+V} = 0.6452$$

$$\therefore F_d = \frac{F_d}{C_V} = 18792.7575 \text{ N}$$

∴  $F_d < F_s$  <sub>revised</sub> ∴ design is safe w.r.t. bending <sup>failure</sup> strength

ii) Wear strength: [calculated for pinion only, bcoz pinion is weaker w.r.t. wear failure  $\because N_1 > N_2$ ]

$$F_w = D_1 Q K b$$

$$Q = \frac{2 G}{G_1 + 1} = \frac{3}{2}$$

$$K = \frac{(\sigma_{es})^2}{1.4} \sin \phi \left[ \frac{1}{E_1} + \frac{1}{E_2} \right]$$

$$= 1.31922 \text{ MPa}$$

$$F_w = 105 \times \frac{3}{2} \times k \times 38 = 20362.16923 \text{ N}$$

$\because F_d < F_w \therefore$  design is safe w.r.t. wear failure.

\* Expression for module [m]:

$$[F_s]_{W.G_1} = ([\sigma_b] Y)_{W.G_1} \cdot b \cdot m$$

$$(F_t)_{\max} \leq (F_s)_{W.G_1}$$

$$\frac{2[T_1]}{D_1} \leq ([\sigma_b] Y)_{W.G_1} \Psi(m) (m) \quad \because b = 4 \text{ m}$$

$$\frac{2[T_1]}{m z_1} \leq ([\sigma_b] Y)_{W.G_1} \Psi(m) (m)$$

$$m \geq \frac{2[T_1]}{([\sigma_b] Y) \Psi z_1}$$

$$m \geq 1.26 \sqrt[3]{\frac{[T_1]}{([\sigma_b] Y) \Psi z_1}}$$

- Det. of power transmitted by spur gear when dimensions are known :
- I/p data:  $m$ ,  $z_1$ ,  $\phi$ ,  $y$ ,  $\psi$  or  $b$ ,  $[\sigma_b] = \text{MPa}$ ,  $C_V$ ,  $C_S$  and  $N \text{ rpm}$
- 1)  $F_S = ([\sigma_b] Y)_{W_G} b m$ .
- $([\sigma_b] Y)_{W_G} = \min \text{ of } ([\sigma_b]_1 Y_1 \text{ and } [\sigma_b]_2 Y_2)$ .
- 2)  $F_d \leq F_S$ .
- $\left( \frac{F_t}{C_V} \right) \text{ or } (F_t \times C_V) = F_S \quad \text{get } F_t = \text{_____ kN}$
- 3) Power ( $P$ ) =  $F_t \times V = \text{_____ W}$
- 4) Rated power =  $P \times C_S = \text{_____ Watts}$ .
- 5)  $F_w = D_1 Q K_b = \text{_____}$   
 $F_w \geq F_d$   
 $\therefore \text{dim ns are safe w.r.t. wear failure.}$

Q Following data is given for spur gear :

- ① No. of teeth on pinion,  $z_1 = 30$ .
- ② " " " gear,  $z_2 = 60$ .
- ③ Speed of pinion,  $N_1 = 1440 \text{ rpm}$ .
- ④ Pressure angle,  $\phi = 20^\circ \text{ F-D}$ .
- ⑤ Module,  $m = 3 \text{ mm}$ .
- ⑥ Face width,  $b = 33 \text{ mm}$ .
- ⑦ Velocity factor,  $C_V = \frac{3}{3+v}$
- ⑧ Lewis form factor,  $y = 0.154 - \frac{0.912}{z}$ .
- ⑨ BHN = 200. both are made of steel with  $S_{UT} = 560 \text{ MPa}$ .
- ⑩ Young's modulus,  $E = 200 \text{ GPa}$ .

Find rated power on the basis of bending failure if  $FoS = 1.5$  and wear strength.

soln  $[\sigma_b] \text{ per bending stress} = \frac{560}{1.5} = \frac{1120}{3}$

$\therefore$  Both gear and pinion are made of same material  
 $\therefore$  pinion will be weaker wheel.

$$F_S = \pi \frac{1120}{3} \left[ 0.154 - \frac{0.912}{30} \right] \times 33 \times 3 = 14351.59949 \text{ N}$$

$$\frac{2 \times 458366.2361}{16 \times 20.6874 \times 10} \leq m^3$$

$$\therefore m \geq 6.52 \approx \boxed{m=7}$$

⑤ dimensions of the gear train:

$$D_1 = m z_1 = 112 \text{ mm} ; \quad D_2 = m z_2 = 336 \text{ mm} ; \quad b = 10 \text{ m} = 70 \text{ mm.}$$

⑥  $F_s = \text{beam strength of weaker gear}$

$$F_s = ([\sigma_b] Y)_{w_G} b \cdot m = 10136.826 \text{ N} \quad \text{--- (1)}$$

⑦  $F_d = \text{dynamic load}$ :

$$F_t = \frac{2 [T_1]}{D_1} = 8185.1113 \text{ N}$$

$$\text{Pitch line velocity} ; V_1 = V_2 = V = \frac{\pi D_1 N_1}{60} = 1.7593 \text{ m/s}$$

$$\text{Velocity Factor, } C_V = \frac{4.5}{4.5 + V} = 0.7189.$$

$$\therefore F_d = \frac{F_t}{C_V} = 11385.1113 \text{ N.}$$

$\because F_d > F_s \therefore \text{design fails w.r.t. to the beam strength of gear.}$

⑧ Revising the module: (by equating  ~~$F_d = F_s$~~ ).

Using:  $F_s = ([\sigma_b] Y)_{w_G} \times 10 \text{ m}^2 \quad (\text{eq (1)})$

$$11385.1113 = 20.6874 \times 10 \times m^2$$

$$m = 7.42$$

$$\boxed{m \approx 8}$$

⑨ Revising the dimensions of the gear train:

$$D_1 = m z_1 = 128 \text{ mm} ; \quad D_2 = 384 \text{ mm} ; \quad b = 10 \text{ m} = 80 \text{ mm.}$$

⑩ Revised beam strength of weaker gear

$$F_s = 20.6874 \times 80 \times 8 = 13239.936 \text{ N.}$$

⑪ Revised dynamic load:

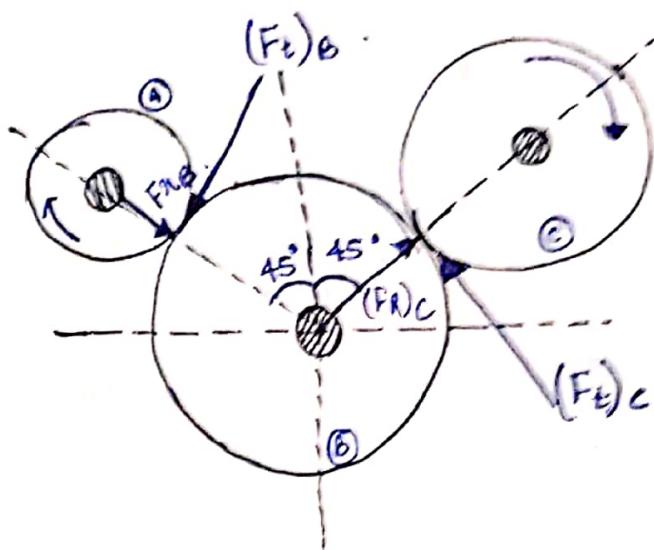
$$F_t = 7161.9724 \text{ N}$$

$$V_1 = \frac{\pi D_1 N_1}{60} = 2.0106 \text{ m/s.}$$

$$\text{Velocity factor} ; C_V = \frac{4.5}{4.5 + V} = 0.6912.$$

$$F_d = \frac{F_t}{C_V} = 10361.9724 \text{ N.}$$

$\therefore F_d < F_s$ : design is safe.



$$V = 2(F_t + F_r) \cos 45^\circ$$

$$H = 0$$

$$(F_R)_2 = (\sqrt{2})(F_t + F_r)$$

For Gear A.

$$T_A = \frac{60 \times 10^6 \times P}{2\pi N_A} = 47746.42293 \text{ N-mm}$$

$$F_t = \frac{2T_A}{D_A} = 636.6198 \text{ N}$$

$$F_r = F_t \tan \varphi = 231.7106 \text{ N}$$

For Gear B

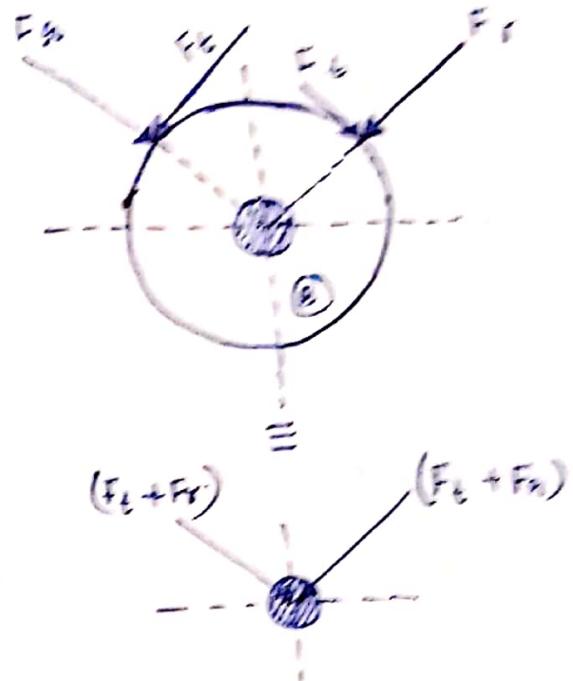
$$T_B = 0$$

For Gear C

$$\frac{T_A}{T_C} = \frac{z_A}{z_C} \Rightarrow T_A = 63661.97724 \text{ N-mm}$$

Now, Resultant force on idler gear shaft:

$$(F_R)_B = \sqrt{2}(F_t + F_r) = 1228 \text{ N}$$



Q1 Given:

① Pressure angle,  $\alpha = 14\frac{1}{2}^\circ$    ② Power transmitted,  $P = 12 \text{ kW}$   
 ③ Speed of pinion,  $N_1 = 3000 \text{ rpm}$    ④ Gear ratio  $i = 3$   
 ⑤ Permissible elastic stress for pinion,  $[\sigma_b]_1 = 105 \text{ MPa}$   
 ⑥ " " " " gear,  $[\sigma_a]_2 = 90 \text{ MPa}$   
 ⑦ Surface endurance limit,  $\sigma_{ES} = 600 \text{ MPa}$   
 ⑧ No. of teeth on pinion,  $z_1 = 16$   
 ⑨ Velocity factor,  $C_V = \frac{4.5}{4.5 + V}$   
 ⑩ Form factor,  $y = 0.124 - \frac{0.684}{z}$   
 ⑪ Young's modulus for steel pinion,  $E_1 = 200 \text{ GPa}$   
 " " " " CI gear,  $E_2 = 100 \text{ GPa}$   
 ⑫ Assuming a service factor,  $C_S = 1.2$ .

Let:  
 ⑬ Face width  $rb = 10 \text{ mm}$   
 i.e.  $\varphi = 10^\circ$

lets.

$$\textcircled{13} \text{ Face width, } b = 10 \text{ m}$$

i.e.  $4p = 10$

### ★ design procedure:

Let subscripts 1 and 2 are used for pinion and gear respectively

①  $T_1$  = mean torque to be transmitted by pinion :

$$T_1 = \frac{60 \times 10^6 \cdot 9}{2\pi N_1} = 381971.8634 \text{ N-mm}$$

$$\textcircled{2} \quad \underline{[T_1] = \text{design torque for pinion}} : \quad [T_1] = T_r \times C_S \\ = 458366.2361 \text{ N-mm}$$

③ No. of teeth on gear :  $Z_2 = G_1 Z_1 = 48$

#### ④ Calculation of module:

↳ Finding the weaker wheel:

$$\text{Pinnen: } \left[ \left[ \sigma_b \right], y_1 \right] = 105 \times \pi \times \left( 0.124 - \frac{0.684}{z_1} \right) = 26.8017 \text{ MPa}$$

$$\text{Gear : } ([\sigma_b]_2)_{4_2} = 60 \pi \left[ 0.124 - \frac{0.684}{2^2} \right] = 20.6874 \text{ MPa}$$

$$\therefore ([\sigma_b]_2 y_2) < ([\sigma_b]_1 y_1)$$

∴ Gear is a weaker wheel. Hence design will be based upon gear.

$$\text{i.e. } ([\sigma_b]_1 Y)_{W.G_1} = ([\sigma_b]_2 Y_2) = 20.6874 \text{ MPa}$$

Hence, module should be calculated using the beam strength of gear

$$[Fs]_{WG_1} = [(\sigma_b)^Y]_{WG_1} \times b \times m$$

$$\frac{2T_1}{m^2} \leq [(\sigma_b)Y]_{wG} \times 10^{-m^2}$$

$$\frac{2 \times 450366.2361}{16 \times 20.6874 \times 10} \leq m^3$$

$$\therefore m \geq 6.52 \approx \boxed{m=7}$$

⑤ dimensions of the gear train:

$$D_1 = m z_1 = 112 \text{ mm} ; \quad D_2 = m z_2 = 336 \text{ mm} ; \quad b = 10 \text{ m} = 70 \text{ mm.}$$

⑥  $F_s = \text{beam strength of weaker gear}$

$$F_s = ([\sigma_b] Y)_{W_G} b \cdot m = 10136.826 \text{ N} \quad \text{--- (1)}$$

⑦  $F_d = \text{dynamic load:}$

$$F_t = \frac{2 [T_1]}{D_1} = 8185.1113 \text{ N}$$

$$\text{Pitch line velocity} ; V_1 = V_2 = V = \frac{\pi D_1 N_1}{60} = 1.7593 \text{ m/s}$$

$$\text{Velocity Factor, } C_V = \frac{4.5}{4.5 + V} = 0.7189.$$

$$\therefore F_d = \frac{F_t}{C_V} = 11385.1113 \text{ N.}$$

$\because F_d > F_s \therefore \text{design fails w.r.t. to the beam strength of gear.}$

⑧ Revising the module: (by equating  ~~$F_d = F_s$~~ ).

Using:  $F_s = ([\sigma_b] Y)_{W_G} \times 10 \text{ m}^2 \quad (\text{eq (1)})$

$$11385.1113 = 20.6874 \times 10 \times m^2$$

$$m = 7.42$$

$$\boxed{m \approx 8}$$

⑨ Revising the dimensions of the gear train:

$$D_1 = m z_1 = 128 \text{ mm} ; \quad D_2 = 384 \text{ mm} ; \quad b = 10 \text{ m} = 80 \text{ mm.}$$

⑩ Revised beam strength of weaker gear

$$F_s = 20.6874 \times 80 \times 8 = 13239.936 \text{ N.}$$

⑪ Revised dynamic load:

$$F_t = 7161.9724 \text{ N}$$

$$V_1 = \frac{\pi D_1 N_1}{60} = 2.0106 \text{ m/s.}$$

$$\text{Velocity factor} ; C_V = \frac{4.5}{4.5 + V} = 0.6912.$$

$$F_d = \frac{F_t}{C_V} = 10361.9724 \text{ N.}$$

$\because F_d < F_s$  : design is safe.

⑫ Calculation of wear load: (always for pinion as pinion is weaker w.r.t. wear failure)

$$K = \frac{\sigma_{es}^2 \sin \alpha}{1.4} \left[ \frac{1}{E_1} + \frac{1}{E_2} \right] = 0.9650 \text{ MPa}$$

$$Q = \frac{2G_1}{G_1 + 1} = \frac{3}{2}$$

$$\therefore F_w = D_1 Q K_b = 14834.682$$

$\therefore F_w > F_d \therefore$  design is safe.

## Design of Shafts:

► COMPLETELY REVERSED (Strength criterion can be used)  
 ► FLUCTUATING  
 ► ALTERNATING ] Since two eq<sup>n</sup> are compulsory for these two loads.

### Case 1: Under variable load

(i) Soderberg equation :- (ductile materials)

$$\frac{\sigma_m}{\sigma_{ut}} + \frac{(k_f) \sigma_a}{\sigma_e} = \frac{1}{N} \quad (N_1 = N_2 = N)$$

(ii) Goodman's equation :- (Brittle material)

$$(k_t) \left[ \frac{\sigma_m}{\sigma_{ut}} \right] + (k_f) \left[ \frac{\sigma_a}{\sigma_e} \right] = \frac{1}{N} \quad (N_1 = N_2 = N)$$

where,  $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$  and  $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$  ;

\*  $k_f = 1 + q_f (k_t - 1)$

\*  $\sigma_e = \sigma_e^* K_a K_b K_c$  = endurance limit of a mechanical component (corrected E.L.).

where  $\sigma_e^* = E.L.$  of a std. specimen.  $\rightarrow$  obtained from S-N curve

= E.L. under reversed bending.

= 0.5 Sut for steels.

= 0.4 Sut for CI.

$K_a$  = size factor;  $K_b$  = S.F. factor.

$K_c$  = load factor  $\begin{cases} = 1 & \text{for reversed bending.} \\ = 0.7 & \text{for reversed axial load.} \\ = 0.6 & \text{for reversed torsion.} \end{cases}$

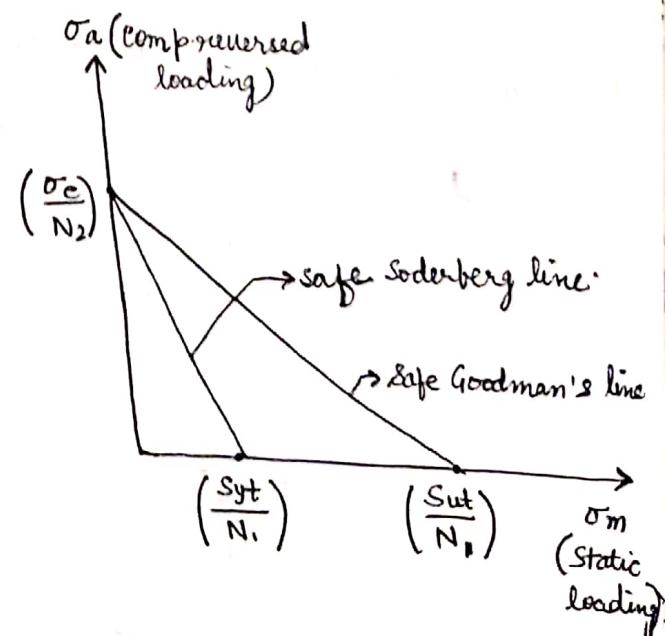
Soderberg eq<sup>n</sup> (Actual) :

$$(k_t) \frac{\sigma_m}{\sigma_{ut}} + (k_f) \frac{\sigma_a}{\sigma_e} = 1.$$

1 for ductile.  $\left( \frac{Syt}{N_1} \right) \quad \left( \frac{\sigma_a}{N_2} \right)$

Goodman's eq<sup>n</sup> (Actual) :

$$(k_t) \frac{\sigma_m}{\left( \frac{\sigma_{ut}}{N_1} \right)} + (k_f) \frac{\sigma_a}{\left( \frac{\sigma_e}{N_2} \right)} = 1$$



- \* Soderberg, Goodman and Gerber's eq<sup>n</sup> are compulsory for design of component under alternating and fluctuating fatigue load. because failure stress of material is unknown when mean and variable stresses are non zero.
- \* Strength criterion, Soderberg, Goodman and Gerber's eq<sup>n</sup> will give same result under completely reversed fatigue loading. [ $\sigma_m = 0$ ,  $\sigma_V = \sigma_{max}$ ] because failure stress of a material [i.e.  $\sigma_{fs}$ ] is known. Hence Soderberg, Goodman and Gerber's eq<sup>n</sup> are optional under completely reversed fatigue loading.
- \* When  $\sigma_m \neq 0$  all the three eq<sup>n</sup> will give different results, hence best eq<sup>n</sup> should be selected for safe design.

### \* Soderberg's equation:

↳ When variable A.L. or variable B.M. alone :-

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + k_f \frac{\sigma_a}{\sigma_e} \quad \text{--- (1)}$$

↳ When variable T.M. acts alone :-

$$\frac{1}{N} = \frac{T_m}{T_{ys}} + k_f \frac{T_a}{T_e} \quad \text{--- (2)}$$

↳ When more than one variable load is acting :-

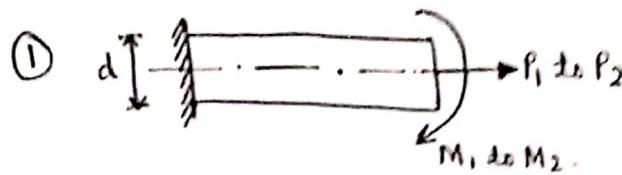
$$\frac{Syt}{N} = \sigma_m + k_f \frac{\sigma_a Syt}{\sigma_e} \quad \text{From eq(1)}$$

$$\therefore \boxed{\sigma_{eq} = \sigma_m + k_f \frac{\sigma_a \sigma_{yt}}{\sigma_e}} \quad \text{--- (3)}$$

For T.M. :-

$$\frac{Syt \text{ or } T_{ys}}{N} = T_m + \frac{k_f T_a T_{ys}}{T_e}$$

$$\boxed{T_{eq} = T_m + \frac{k_f T_a T_{ys}}{T_e}} \quad \text{--- (4)}$$



From eq. III:

$$(\sigma_{eq})_a = \frac{4P_m}{\pi d^2} + k_f \frac{4P_a}{\pi d^2} \cdot \frac{\sigma_{yt}}{\sigma_e}$$

$$(\sigma_{eq})_a = \frac{X}{d^2} \text{ MPa} \quad \text{--- (A)}$$

From eq. III:

$$(\sigma_{eq})_b = \frac{32M_m}{\pi d^3} + k_f \left[ \frac{32M_a}{\pi d^3} \right] \left[ \frac{\sigma_{yt}}{\sigma_e} \right]$$

$$(\sigma_{eq})_b = \frac{Y}{d^3} \quad \text{--- (B)}$$

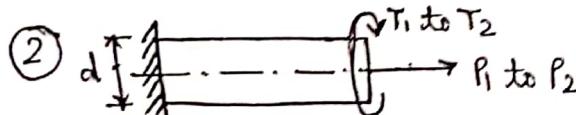
$$\therefore (\sigma_{eq})_R = (\sigma_{eq})_a + (\sigma_{eq})_b = \frac{X}{d^2} + \frac{Y}{d^3} \quad \text{--- (C)}$$

$$(\sigma_{max})_{ind} \leq \sigma_{per}$$

$$\frac{X}{d^2} + \frac{Y}{d^3} \leq \frac{Syt}{N}$$

$$d \geq \text{--- mm}$$

$$(\sigma_{eq})_R \leftarrow \boxed{\text{---}} \rightarrow \sigma_x = (\sigma_{eq})_R$$



From eq. III:  $(\sigma_{eq})_a = \frac{4P_m}{\pi d^2} + k_f \frac{4P_a}{\pi d^2} \cdot \frac{\sigma_{yt}}{\sigma_e}$

$$(\sigma_{eq})_a = \frac{X}{d^2} \text{ MPa} \quad \text{--- (A)}$$

From eq. IV:  $\sigma T_{eq} = \frac{16T_m}{\pi d^3} + k_f \frac{16T_a}{\pi d^3} \left( \frac{T_{ys}}{T_e} \right)$

$T_{eq} = \frac{Z}{d^3} \text{ MPa} \quad \text{--- (B)}$

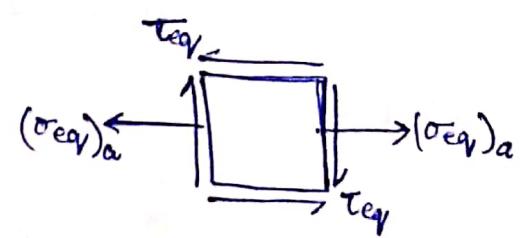
$\frac{Syt}{2}$  MSST  
 $\frac{Syt}{\sqrt{3}}$  MDET

by using MSST:

$$(\sigma_t)_{per} = \sqrt{\sigma_x^2 + 4T_{eqy}^2}$$

$$T_{per} = \frac{1}{2} \sqrt{\sigma_x^2 + 4T_{eqy}^2}$$

$$\frac{Syt}{N} = \frac{1}{2} \sqrt{(\sigma_{eq})_a^2 + 4T_{eqy}^2} \quad ; \quad d = \text{--- mm}$$



by using MDET:

$$(\sigma_t)_{per} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\tau_{per} = \frac{1}{\sqrt{3}} \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

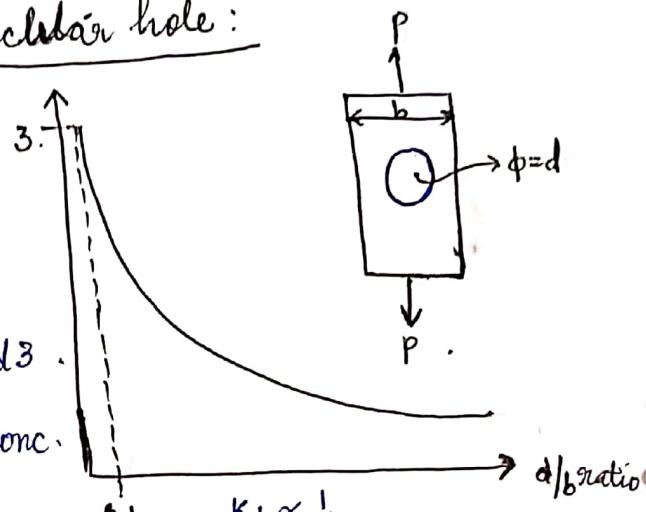
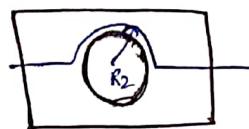
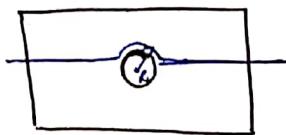
$$\frac{S_{eq}}{N} = \frac{1}{\sqrt{3}} \sqrt{(\sigma_{eq})_a^2 + 3(\tau_{eq})^2}$$

\* stress concentration factor for circular hole:

\* For circular hole,  
max stress conc. factor = 3.  
its actual value depends on  $\frac{d}{b}$  ratio

Hence, stress conc. factor lies b/w 1 and 3.

\* smaller hole will have higher stress conc. factor.



Q A hot rolled steel shaft subjected to torsional moment varies from 300 kN-mm (CCW) to 100 kN-mm (ACW) and bending moment at critical cross section varies from 400 kN-mm to -200 kN-mm. The shaft is of uniform diameter and no keyway is present at the critical x-s/c. Determine diameter of shaft. by taking  $FoS = 1.5$ . assume ultimate, yield and design stress at 560 MPa, 420 MPa and 280 MPa. Modification factor = 0.62; size correction factor = 0.85 load factor for bending = 1. load factor for torsion = 0.58.

So/ln. Given:

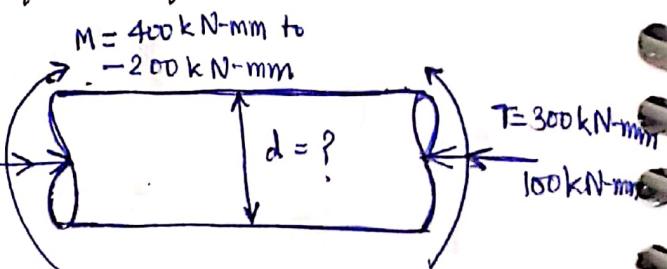
$K_t = K_f = 1$  [because of no discontinuity]

$S_yt = \sigma_{yt} = 420 \text{ MPa}$

$S_{ut} = \sigma_{ut} = 560 \text{ MPa}$

design stress ( $\sigma_e^*$ ) = 280 MPa.

size factor,  $K_a = 0.85$



surface finish convection factor  $k_b = 0.62$

load factor for bending  $K_c = 1$

load factor for torsion  $K_c = 0.58$

F.O.S.  $N = 1.5$

① Assuming variable bending moment acting alone :-

Soderberg eqn is used because of ductile material (hot rolled steel).

$$M_{max} = 400 \text{ kN-mm}; M_{min} = -200 \text{ kN-mm}$$

$$M_m = \frac{M_{max} + M_{min}}{2} = 100 \text{ kN-mm}; M_a = \frac{M_{max} - M_{min}}{2} = 300 \text{ kN-mm}$$

$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{3200000}{\pi d^3} \text{ MPa}; \sigma_a = \frac{32 M_a}{\pi d^3} = \frac{96 \times 10^5}{\pi d^3} \text{ MPa}$$

$$\sigma_e = \sigma_e^* K_a K_b K_c = 280 \times 0.85 \times 0.62 \times 1 = 147.56 \text{ MPa}$$

Hence, from Soderberg's eqn:

$$\sigma_{eq} = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e} = \frac{30.5245 \times 10^6}{\pi d^3} \text{ MPa} \quad \text{--- (1)}$$

② Assuming variable twisting moment alone :-

$$T_{max} = 300 \text{ kN-mm}; T_{min} = -100 \text{ kN-mm}$$

$$T_m = \frac{T_{max} + T_{min}}{2} = 100 \text{ kN-mm}; T_a = \frac{T_{max} - T_{min}}{2} = 200 \text{ kN-mm}$$

$$T_m = \frac{16 T_m}{\pi d^3} = \frac{16 \times 10^5}{\pi d^3} \text{ MPa}; T_a = \frac{16 T_a}{\pi d^3} = \frac{32 \times 10^5}{\pi d^3} \text{ MPa}$$

$$T_e = \sigma_e^* (K_a) (K_b) (K_c) = 280 \times 0.85 \times 0.62 \times 0.58 = 85.5848 \text{ MPa}$$

Using Soderberg's eqn:-

$$T_{eq} = T_m + K_f \frac{T_a T_{yt}}{\sigma_e}$$

[Using MSST,  $T_{yt} = \frac{S_{yt}}{2}$ ]

$$T_{eq} = \frac{9.4513 \times 10^6}{\pi d^3} \text{ MPa}$$

$$T_{yt} = \frac{420}{2}$$

③ Diameter of shaft: (d)

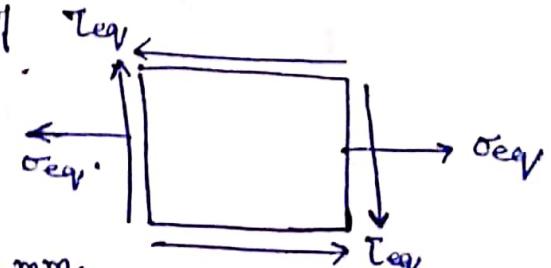
Diameter is determined by using MSST because of biaxial state of stress and ductile material.

$$T_{per} = \sqrt{(\sigma_{eq})^2 + 4(T_{eq})^2}$$

$$\frac{420}{2 \times 1.5} = \frac{35.9040 \times 10^6}{\pi d^3 \times 2}$$

$$d = 34.433798 \text{ mm.}$$

$d \approx 35 \text{ mm.}$  [standard dia. in series of 5]



Q For the stepped bar as shown determine for an infinite life, assume  $K_a = 0.9$ ,  $K_b = 0.8$ ,  $K_{ct} = 1.5$ ,  $\gamma = 0.9$ ;  $S_{yt} = 250 \text{ MPa}$ ,  $S_{ut} = 400$   $F.O.S = 2$

$$\frac{[(\sigma_b)_{max}]_B}{[(\sigma_b)_{max}]_A} = \frac{(M_{max})_B}{(M_{max})_A} \cdot \left[ \frac{d_A}{d_B} \right]^3$$

$$= \frac{1}{3} [\theta] = \frac{8}{3}.$$

$$\frac{(\sigma_{axial})_B}{(\sigma_{axial})_A} = \left( \frac{d_A}{d_B} \right)^2 = 4.$$

Hence, critical x-s/c is the x-s/c where dis continuity present. i.e. at B.

$$[\because (\sigma_b)_B = \frac{8}{3} (\sigma_b)_A \text{ and } (\sigma_a)_B = 4 (\sigma_a)_A]$$

\* Stress analysis should be carried at x-s/c 'B'.

① When 'W' is acting alone:

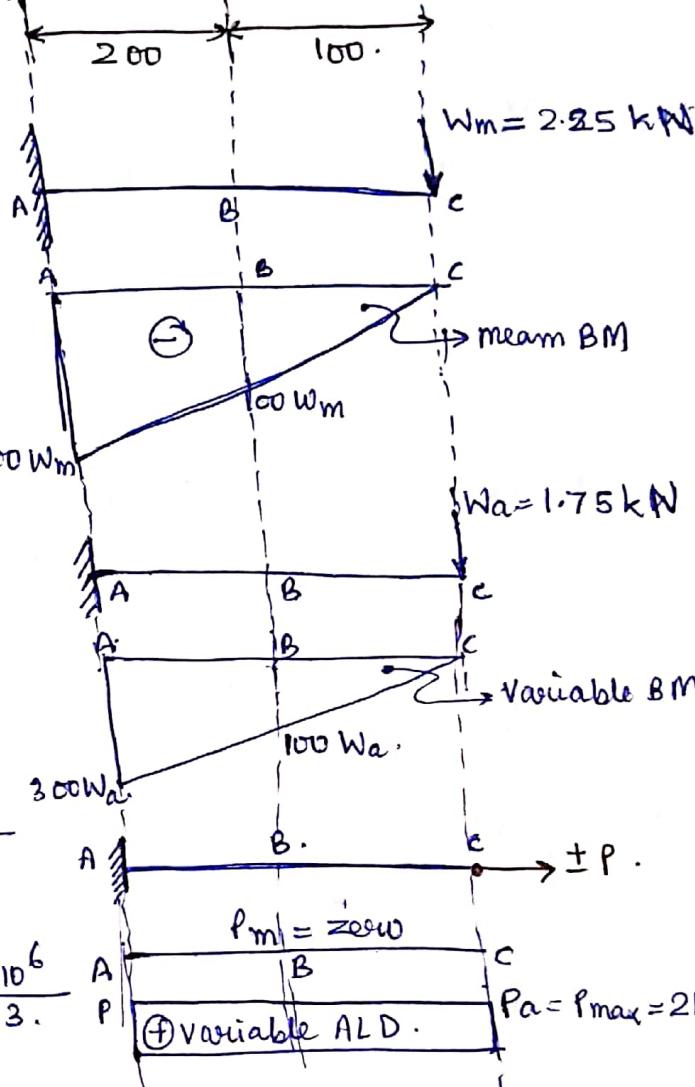
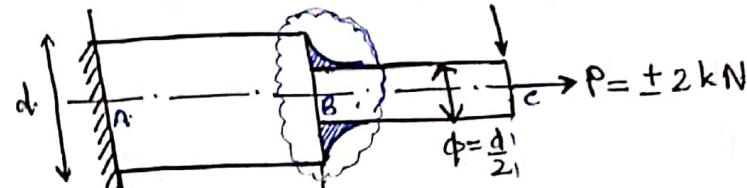
$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{32 \times 100 \times 2.25 \times 10^3}{\pi \left(\frac{d}{2}\right)^3} = \frac{57.6 \times 10^6}{\pi d^3}$$

$$\sigma_a = \frac{32 M_a}{\pi d^3} = \frac{32 \times 100 \times 1.75 \times 10^3}{\pi \left(\frac{d}{2}\right)^3} = \frac{44.8 \times 10^6}{\pi d^3}$$

$$K_f = 1 + \gamma (K_f - 1) = 1.45.$$

$$\sigma_e = (\sigma_e^*) (K_a \cdot K_b \cdot K_c) = 144 \text{ MPa.}$$

critical x-s/c  $W = 0.5 \text{ to } 4 \text{ kN}$



$$\sigma_e^* = 50\% \text{ of } S_{ut} \\ = 200$$

by using Soderberg's eqn:

$$(\sigma_{eq})_b = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e} = \frac{17.2629 \times 10^6}{\pi d^3} \text{ MPa}$$

When completely reversed axial load (P) acts:

$$\sigma_m = 0$$

$$\sigma_a = \sigma_{max} = \frac{4 P_a}{\pi d_B^3} = \frac{324 \times 10^3}{\pi d^2}$$

$$(\sigma_{eq})_a = 0 + K_f \frac{\sigma_a \sigma_{gt}}{\sigma_e} = \frac{11.6599 \times 10^3}{\pi d^2} \text{ MPa.}$$

$$\sigma_e = \sigma_e^* K_a K_b K_C = 100.8 \text{ MPa}$$

$$K_C = 0.7.$$

∴ diameter of bar (d):

$$(\sigma_{max})_{ind} \leq \sigma_{per.}$$

$$\frac{17.2629 \times 10^6}{\pi d^3} + \frac{11.6599 \times 10^3}{\pi d^2} \leq \frac{250}{2}$$

$$d \geq 76.994 \text{ mm}$$

$$d = 77 \text{ mm.}$$

$$\sigma_x \leftarrow \boxed{\quad} \rightarrow \sigma_x = (\sigma_{eq})_b + (\sigma_{eq})_a$$

Fig. State of stress at critical plane (i.e. top fibre of critical x-s/c (B)).

Q. The non-rotating shaft is subjected to load P varying 4kN to 12kN.

$Sut = 600 \text{ MPa}$ ;  $Syt = 350 \text{ MPa}$ ; Endurance limit = 300 MPa;

size factor = 0.8 ~~1.0~~, surface finish factor = 0.85,  $q_f = 0.9$ ;

$FoS = 3.5$ ,  $K_f = 1.8$ . Find the diameter D using Goodman's eq<sup>n</sup>:

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_C} = \left[ \frac{M_B}{M_C} \right] \left[ \frac{dc}{dB} \right]^3$$

mean  $\sigma_b$ : -

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_C} = \frac{240}{240} \times 2^3 = 4.8.$$

Variable  $\sigma_b$ : -

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_C} = \frac{120}{120} \times 2^3 = 4.8$$

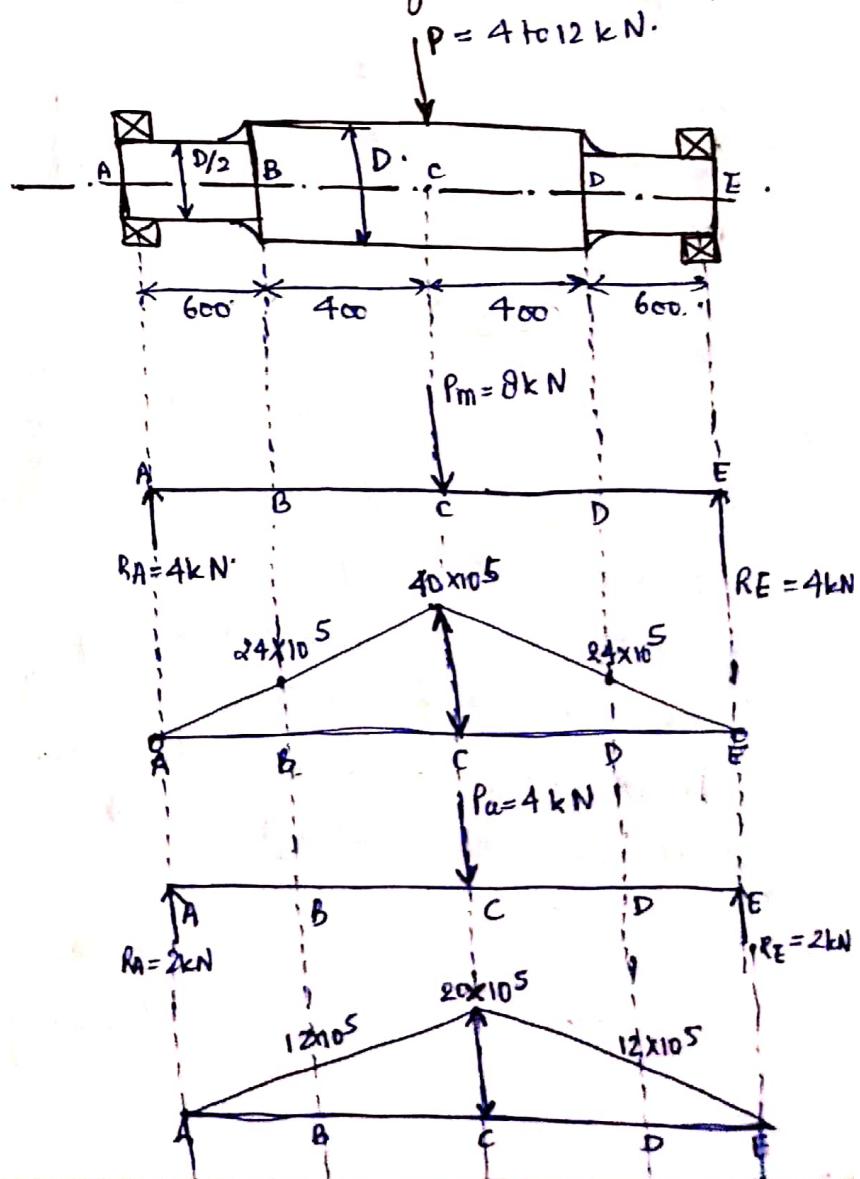
At critical x-s/c:

$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{32 \times 24 \times 10^5 \times 0}{\pi D^3}$$

$$= \frac{614.4 \times 10^6}{\pi D^3}$$

$$\sigma_v = \frac{32 M_a}{\pi d^3} = \frac{32 \times 12 \times 10^5 \times 0}{\pi D^3}$$

$$= \frac{307.2 \times 10^6}{\pi D^3}$$



$$K_f = 1 + q(K_f - 1) = 1.72.$$

$$\sigma_e = (\sigma_e^*) (K_a) (K_b) (K_c) = 204 \text{ MPa}.$$

Using Goodman's eqn:

$$\frac{1}{N} = K_f \left( \frac{\sigma_m}{\sigma_{ut}} \right) + K_f \left( \frac{\sigma_a}{\sigma_e} \right).$$

$$\frac{1.8432 \times 10^6}{\pi d^3} + \frac{2.5901 \times 10^6}{\pi d^3} = \frac{1}{3.5}$$

$$d = 170.3 \text{ mm}$$

$$d \approx 175 \text{ mm}.$$

Q For the rotating shaft as shown in fig. Determine the life of shaft.

$S_{ut} = 500 \text{ MPa}$ ; endurance limit of shaft is 200 MPa.

$K_f$  = fatigue stress conc. factor = 2.

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_c} = \frac{M_B}{M_c} \times \left( \frac{d_c}{d_B} \right)^3$$

$$\frac{(\sigma_b)_{B+D}}{(\sigma_b)_c} = \frac{15}{25} \times 2^3 = 4.8$$

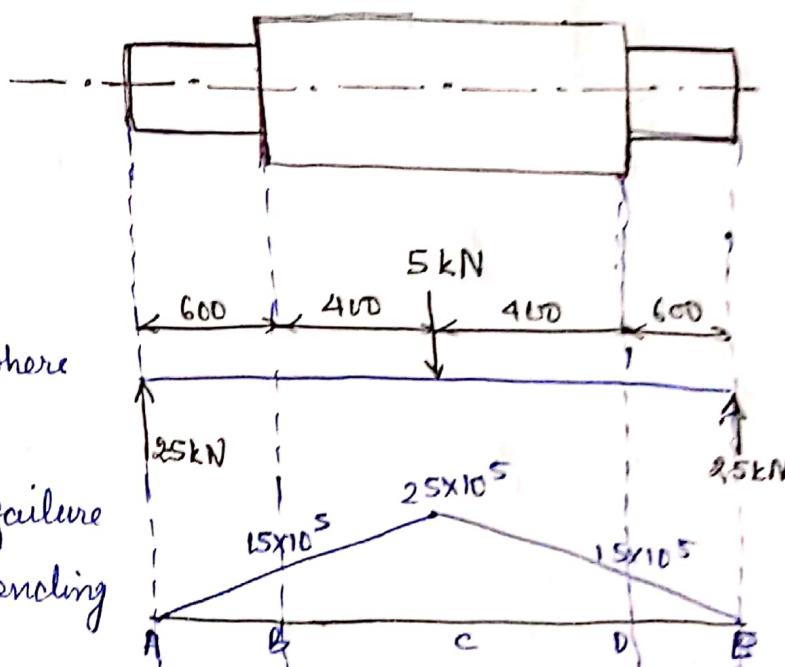
\* Critical x-s/c are B & D (i.e. where fillets are provided)

\* Shaft is subjected to fatigue failure because completely reversed bending stresses are developed at the

critical points (i.e. on bottom fibres) on the critical x-s/c due to rotation of shaft.

fatigue loading

failure is under tension only  
(rarely in compression)



### Critical cross-section:

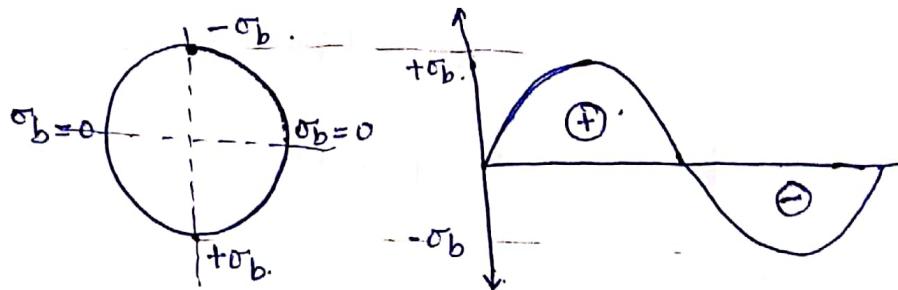


Fig. Stress cycle for comp. reversed bending stress.

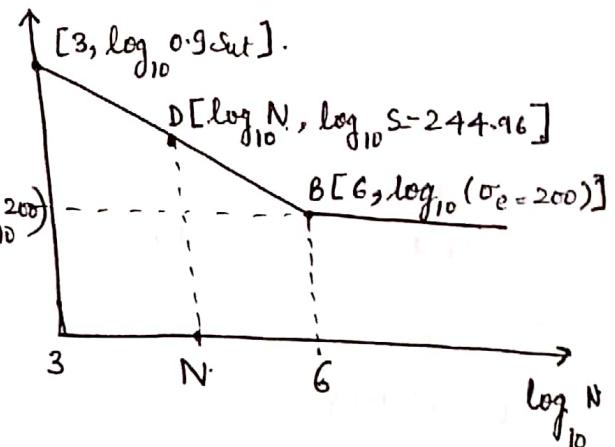
$$(\sigma_{\max}) = K_f (\sigma_b)_{\max} = 2 \times \frac{32M}{\pi (50)^3} = 244.96 \text{ MPa} > \sigma_e.$$

Hence, shaft will have finite life.

$$\frac{\log_{10} (0.9 \times 500) - \log_{10} (200)}{\log_{10} (244.46) - \log_{10} (200)} = \frac{6-3}{6 - \log_{10} N} \log_{10} (3, \log_{10} 200)$$

$$N = 180868.1 \text{ cycles.}$$

$$N = 180869 \text{ cycles.}$$



\* If diameter is 75 mm instead of 50 mm shaft will have infinite life because the max stress is less than σ\_e.

$$\text{i.e. } \sigma_{\max} = 72.43 < 200 \text{ MPa.}$$

Q A m/c component is subjected to biaxial state of stress as shown. Determine FOS by using MDET. Assume,  $Syt = 250 \text{ MPa}$ ,  $Sut = 500 \text{ MPa}$

$K_t = 1.85$ ;  $q_f = 0.9$ ; corrected endurance limit = 200 MPa.

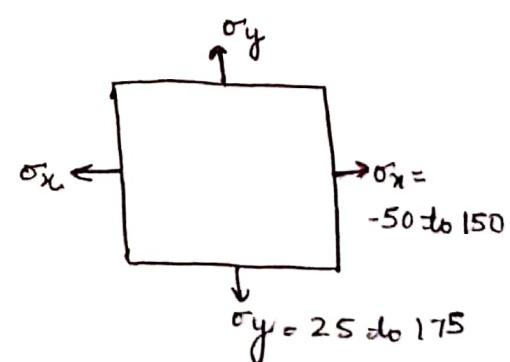
$$(\sigma_x)_m = \frac{150 - 50}{2} = 50 \text{ MPa.}$$

$$(\sigma_x)_a = \frac{150 + 50}{2} = 150 \text{ MPa.}$$

$$K_f = 1 + q_f (K_t - 1) = 1.765.$$

$$\sigma_e = \sigma_e * K_a K_b K_c = 200 \text{ MPa.}$$

$$(\sigma_x)_{eq} = \sigma_m + K_f \frac{\sigma_a \sigma_{yt}}{\sigma_e} = 270.625 \text{ MPa.}$$



$$(\sigma_y)_{\text{mean}} = 100 \text{ MPa}; \quad (\sigma_y)_a = 75 \text{ MPa.}$$

$$(\sigma_y)_{\text{eq}} = 100 + 1.765 \times \frac{75 \times 250}{200} = 265.46875 \text{ MPa.}$$

$$\sigma_1 = 270.625 \quad [\text{Larger of } (\sigma_x)_{\text{eq}} \text{ & } (\sigma_y)_{\text{eq}}].$$

$$\sigma_2 = 265.46875 \text{ MPa.}$$

by using MDET

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 \leq \left( \frac{S_y t}{F_{\text{OS}}} \right)^2$$

$$\underline{F_{\text{OS}} = 0.9325}$$

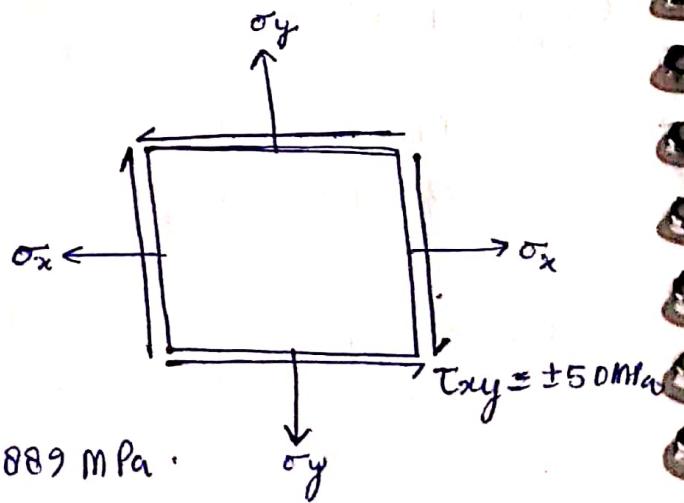
Q If  $T_{xy}$  is also given,

$$T_m = 0$$

$$T_a = 50 \text{ MPa.}$$

$$T_{\text{eq}} = T_m + K_f \frac{T_a T_{sy}}{T_e \rightarrow \sigma_c \times K_c}$$

$$= 0 + \frac{1.765 \times 50 \times 250}{200 \times \sqrt{3}} = 63.6889 \text{ MPa.}$$



Q

$$S_{ut} = 400 \text{ MPa}$$

$$S_{yt} = 250 \text{ MPa}$$

$$k_f = 2; N = 1.5; K_a = 0.85, K_b = 0.7$$

